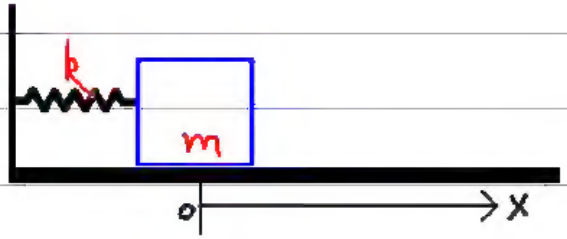
Meriam 8/2: Vibrations (contd.)

Quick Review:



- Select origin at neutral position
- Displace mass a small +ve distance x
- Release the mass

$$\sum F_x = -kx = m\ddot{x} \rightarrow \ddot{x} + \frac{k}{m}x = 0 \longleftrightarrow \ddot{x} + \omega_n^2 x = 0$$

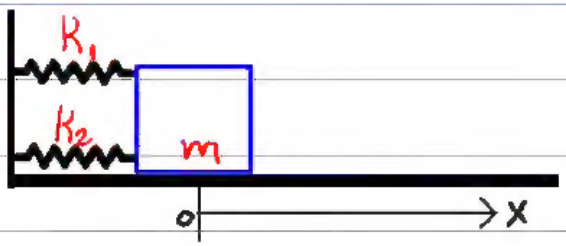
- Solution is of the form:

$$x(t) = \boxed{C_3 \sin(\omega_n t + \phi)}, \quad \omega_n = \sqrt{k/m}$$

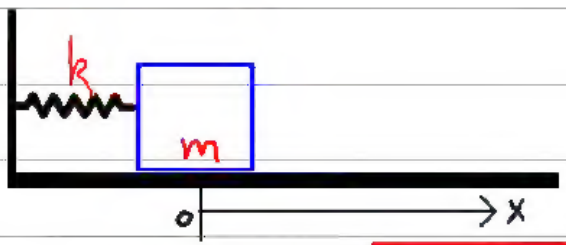
- If required, get C_3 & ϕ from initial conditions

Consider more complex configurations: 2 parallel springs
(ignore rotational effects)

$$\sum F_x = -K_1 x - K_2 x = m\ddot{x}$$
$$-(K_1 + K_2)x = m\ddot{x}$$

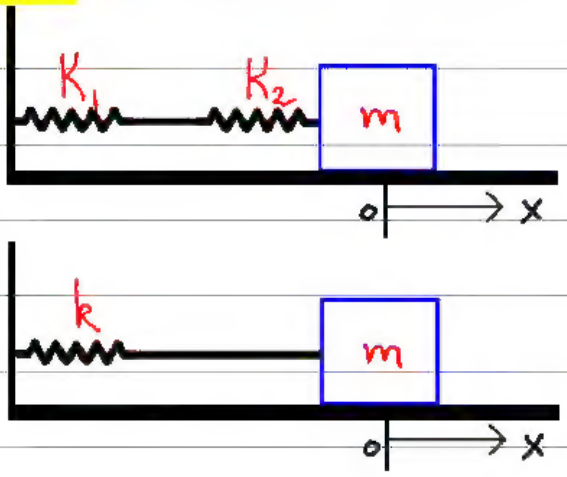


$$\sum F_x = -kx = \ddot{x}$$



→ Replace parallel springs K_1 & K_2 with a single spring $\boxed{k = K_1 + K_2}$

Consider 2 springs in series



- Set origin at neutral position
- Displace mass a small distance x
- $x = x_1 + x_2$, where

  x_1 = extension of K_1

  x_2 = extension of K_2

→ Tension in both springs & connecting rope is $K_1 x_1 = K_2 x_2$

$$\rightarrow \boxed{\frac{K_1}{K_2} = \frac{x_2}{x_1}}$$

We seek a single spring of strength k that has "equivalent" stiffness

ie. $\quad k x = K_1 x_1 \quad (= K_2 x_2)$

$$\rightarrow k = \frac{K_1 x_1}{x} = \frac{K_1 x_1}{x_1 + x_2} = \frac{K_1}{1 + x_2/x_1}$$

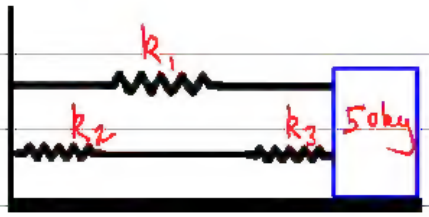
Substitute $\frac{K_1}{K_2} = \frac{x_2}{x_1}$ → $k = \frac{K_1}{1 + K_1/K_2}$

$$\rightarrow \boxed{k = \frac{K_1 K_2}{K_1 + K_2}}$$ for 2 springs in series.

Mechanical/Electrical Analogs

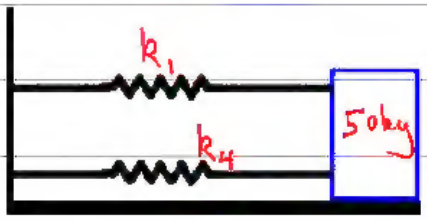
	Resistors	Capacitors	Springs
Series	$R = R_1 + R_2$	$C = \frac{C_1 C_2}{C_1 + C_2}$	$k = \frac{k_1 k_2}{k_1 + k_2}$
parallel	$R = \frac{R_1 R_2}{R_1 + R_2}$	$C = C_1 + C_2$	$k = k_1 + k_2$

Eg. Find ω_n

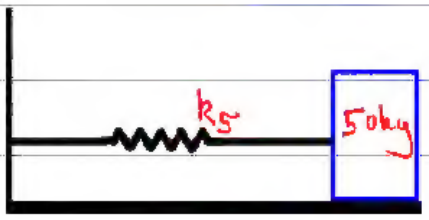


Given:

$k_1 = 1000\ N/m$
$k_2 = 3000\ N/m$
$k_3 = 6000\ N/m$



$k_4 = \frac{k_2 k_3}{k_2 + k_3}$
$= 2000\ N/m$



$k_5 = k_1 + k_4$
$= 3000\ N/m$

$\omega_n^2 = k_5/m = 3000\ N/m / 50kg = 60\ s^{-2}$

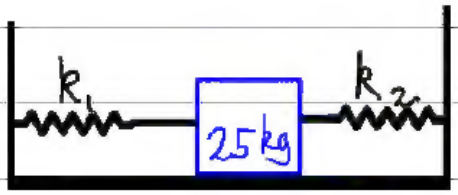
$\omega_n = 7.75\ s^{-1}$

More complex geometries.

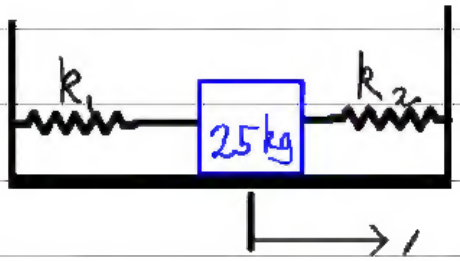
E.g. Find ω_n, $x(t)$

$k_1 = 3000\ N/m$

$k_2 = 4,800\ N/m$



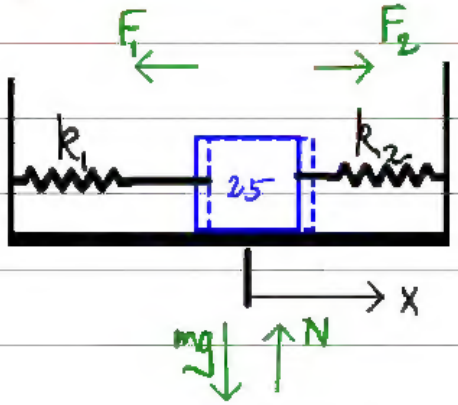
Where is neutral point? where should I place origin?
Does it matter? (No!)



1/ Set origin at neutral position

2/ Displace the mass a small +ve distance x, & release

3/ Determine ΣF_x what has changed due to small x movement?



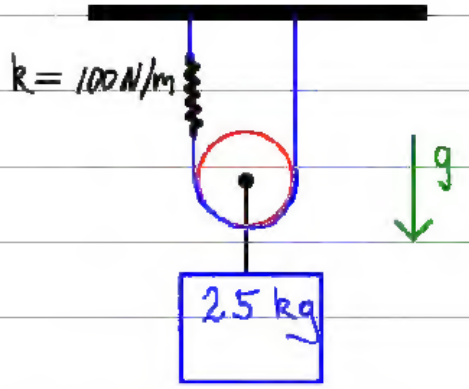
$$\Sigma F_x = -k_1 x - k_2 x$$
$$= -3,000\,x - 4,800\,x$$

$$\Sigma F_x = -7,800\,x = 2.5\,\ddot{x}$$

$$\rightarrow \ddot{x} + 312\,x = 0 \rightarrow \omega_n^2 = 312$$

$$x(t) = \text{Constant} \cdot \sin(\sqrt{312}\,t + \phi)$$

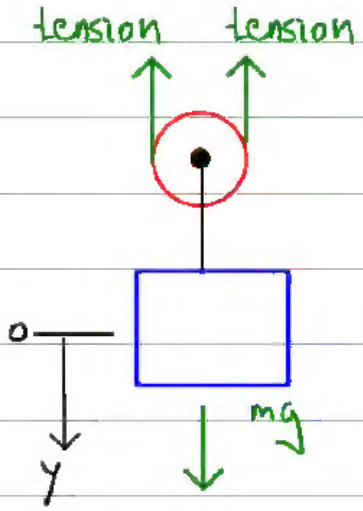
Eg. Find ω_n.



1/ Draw FBD

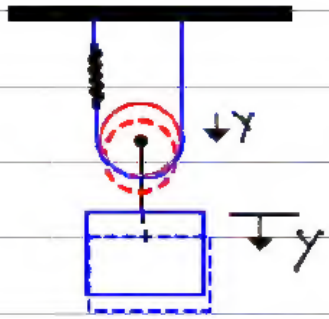
Identify axis ↓y

Identify origin as neutral point.



original tension = mg/2

2/ Displace mass a small +ve distance y, & release



→ pulley moves dow a distance y

→ rope/spring system elongates by 2y

→ spring tension increases by 2ky

3/ $\Sigma F_y = -$ (left side of pulley + right side of pulley) (2ky)

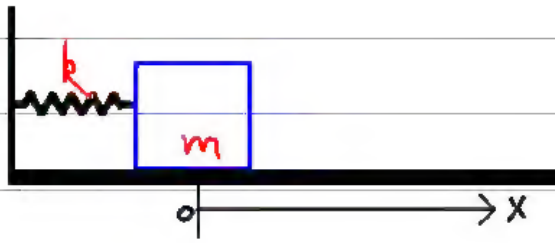
$= -(2)(2ky)$

$\Sigma F_y = -4ky = m\ddot{y}$, $k = 100 N/m$; $m = 25\ kg$

4/ $\ddot{y} + \omega_n^2 y = 0 \rightarrow \omega_n = \sqrt{4k/m} = \sqrt{16} = 4\ s^{-1}$

Summary

Basic Linear Vibrations



- Select origin at neutral position
- Displace mass a small +ve distance x
- Release the mass

- $\sum F_x = -kx \quad m\ddot{x} \rightarrow \ddot{x} + \frac{k}{m}x = 0 \leftrightarrow \ddot{x} + \omega_n^2 x = 0$

- Solution is of the form:

$$x(t) = \boxed{C_3 \sin(\omega_n t + \phi)}, \omega_n = k/m$$

- Springs in series and parallel
- Technique to solve novel configurations

Next Lecture

- Rotational vibrations
- Damped vibrations

} (Meriam 8/2)